Study Guide for Unit 4 Lesson 4 and 5

Lesson 4-Number Theory

The number 1 is neither prime nor composite because it has only one factor.

Prime Numbers: A whole number that has only two factors: 1 and itself.

Examples:

Composite Numbers: A whole number that has more than two factors.

Examples:

Practice:

Complete the table.

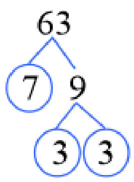
Number	List of Factors	Prime or composite?
10		
5		
12		
18		
41		

Factor Trees

Factors are the numbers multiplied together to obtain a given product. For example, 3 and 5 are factors of 15. 1 and 15 are also factors of 15.

Divisibility Rules			
A number is divisible by. . .		Divisible	Not Divisible
2	if the last digit is even (0, 2, 4, 6, or 8).	3,978	4,975
3	if the sum of the digits is divisible by 3.	315	139
4	if the last two digits form a number divisible by 4.	8,512	7,518
5	if the last digit is 0 or 5.	14,975	10,978
6	if the number is divisible by both 2 and 3	48	20
9	if the sum of the digits is divisible by 9.	711	93
10	if the last digit is 0.	15,990	10,536

Use a factor tree to find the prime factorization of 63.

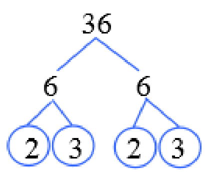


Start by finding two numbers whose product is 63. The number 7 is prime, circle it. The number 9 is not, so find two numbers whose product is 9.

The number 3 is prime, circle both threes. You are finished when the numbers at the bottom of each branch are prime numbers.

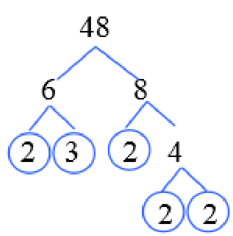
The prime factorization of $63 = 3 \cdot 3 \cdot 7$.

Example 1 **Find the prime factorization of 36.**



The prime factorization of $36 = 2 \cdot 2 \cdot 3 \cdot 3$

Example 2 **Find the prime factorization of 48.**



The prime factorization of $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$

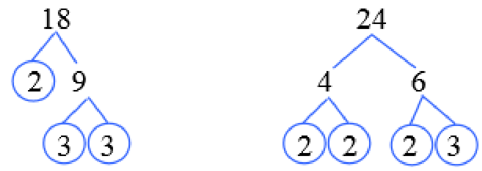
The Greatest Common Factor (GCF)

The **Greatest Common Factor (GCF)** is the largest factor that is the same in all the given numbers. (The largest number that can divide evenly into all the numbers.)

To find the greatest common factor, start by prime factoring each number. Then identify the common factors. If there is more than one common factor, the greatest common factor is the product of all the common factors. If there are no common factors, the greatest common factor is 1.

Example 3

Find the greatest common factor of 18 and 24.



Prime factor each number.

$2 \cdot 3 \cdot 3$ $\quad$ $2 \cdot 2 \cdot 2 \cdot 3$

Write as a product of prime factors from least to greatest.

$\not{2} \cdot \not{3} \cdot 3$ $\quad$ $\not{2} \cdot 2 \cdot 2 \cdot \not{3}$

Identify and write the common factors. (It may be helpful to cross out the common factors.) There are two factors in common.

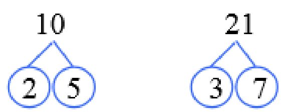
$\text{GCF} = 2 \cdot 3$

$\text{GCF} = 6$

Multiply the common factors to get the greatest common factor.

Example 4

Find the greatest common factor of 10 and 21.



Prime factor each number.

$2 \cdot 5$ $\quad$ $3 \cdot 7$

Write as a product of prime factors from least to greatest.

$\text{GCF} = 1$

Identify and write the common factors. There are no common factors, so the greatest common factor is 1.

The Least Common Multiple (LCM)

The **Least Common Multiple (LCM)** is the smallest number that is a multiple of each of the given number.

We will demonstrate two methods for finding the least common multiple. The first method uses factor tree, the second method uses what is known as repeated division.

Example 5

Find the least common multiple of 4 and 10.

Method 1 – Make a factor tree for each number.

Find the prime factors they have in common. (If you have three or more numbers, your common factors need to appear in at least two of the numbers.)

In our problem, the numbers have a 2 in common.

We will multiply the common factor, 2, along with any numbers that are not in common, in this case 2, and 5. Our least common multiple (LCM) is:

$$LCM = 2 \cdot 2 \cdot 5$$
$$LCM = 20$$

Practice:

Find the Greatest Common Factor (GCF) of the following sets of numbers.

1. 8 and 12
2. 24 and 40
3. 9 and 10
4. 12 and 35

Find the Least Common Multiple (LCM) of the following sets of numbers.

1. 3 and 4
2. 4 and 6
3. 9 and 15
4. 15 and 25

Exponents - An exponent is a shorthand way of writing multiplication of the same number

10^3 10 is the base number. It is read: Ten to the third power. 3 is the exponent. It means: 10 x 10 x 10

(The exponent tells how many times a number should be multiplied by itself)

Example 1: 4^4 = 4 x 4 x 4 x 4 = 256

Example 2: 8 x 8 x 8 x 8 x 8 x 8 = 8^6

Example 3: 5^3 + 2^2 = (5 x 5 x 5) + (2 x 2) = 125 + 4 = 129

Square roots - When a number is a product of 2 identical factors, then either factor is called a square root. A root is the inverse of the exponent.

Example 1: $\sqrt{4}$ = 2

Example 2: $\sqrt{100}$ = 10 T

These are all called perfect squares because the square root is a whole number.

PRACTICE:

Find the value of each expression.

3^8 4^6 7^3 8^4 9^2

Write each product in exponent form.

6 x 6 x 6 x 6 x 6 5 x 5 x 5 x 5 7 x 7 x 7

2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 11 x 11 x 11

Write the square root.

a. $\sqrt{144}$ = __________

b. $\sqrt{81}$ = __________

c. $\sqrt{9}$ = __________

d. $\sqrt{49}$ = __________

e. $\sqrt{100}$ = __________

f. $\sqrt{36}$ = __________

Lesson 5-Problem Solving: Number Theory

Addition

How many in all
Altogether
Sum
Join
Total
Both
Add

Subtraction

Minus	Away
Difference	Remain
Less	Than

How many more
How many are left

"er" words such as longer, farther, fewer, faster

Multiplication

Times
Each
Area
Total
Product
How many in all
Multiples of
Altogether

Division

How many in each
Shared equally
Part
Per
Divided
Quotient
Groups of
Factors

Additional Resources:

http://www.sheppardsoftware.com/mathgames/numbers/fruit_shoot_prime.htm

https://www.khanacademy.org/math/cc-fourth-grade-math/cc-4th-fact-mult-topic/cc-4th-prime-composite/v/recognizing-prime-numbers

http://www.ixl.com/math/grade-4/prime-and-composite-numbers

http://www.xpmath.com/forums/arcade.php?do=play&gameid=60

http://www.mathnook.com/math/skill/primecompositegames.php

http://www.mathplayground.com/factortrees.html

https://www.khanacademy.org/math/pre-algebra/factors-multiples/prime_factorization/v/prime-factorization

http://www.math-play.com/Exponents-Jeopardy/Exponents-Jeopardy.html

http://www.math-play.com/square-roots-game.html